

# Exploring the Addition of Boundary Energy to the Marked Point Process

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# Overview

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- **Motivation**
  - Problem statement: MPP is limited to generic shapes
  - Illustrative dataset: Fiber reinforced composites
- **Marked Point Process (MPP)**
  - Set up: MPP = Point Process + Marks
  - Energy minimization using multiple birth and deaths
- **Part I: MPP and parametric Active Contours(AC)**
  - AC boundary energy: Smoothness and Curvature
  - Combination of MPP-AC: Disks with deformed boundaries
- **Part II: MPP and Level Sets(LS)**
  - LS boundary energy: Dark regions with strong edges
  - Object proposal: LS alone can guide the object proposal
- **Future Work**
  - Deep learning based approaches

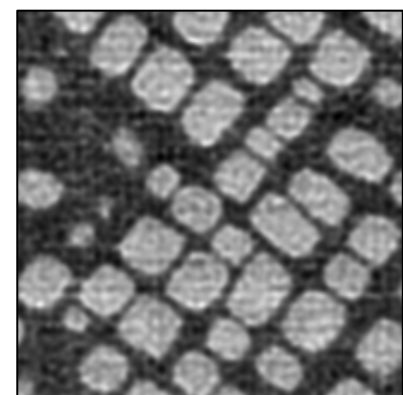
# Motivation

# Introduction

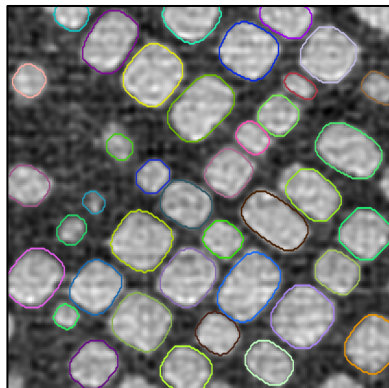
Motivation – Expand the limitations of the Marked Point Process(MPP)

## What is the MPP?

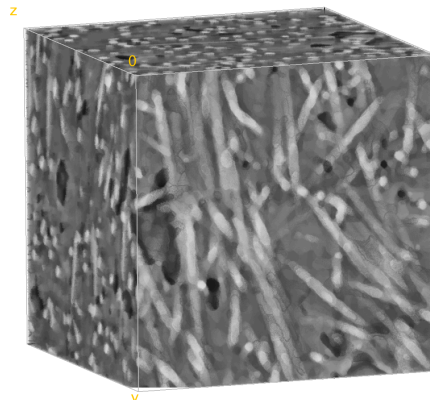
- Stochastic framework that models images as configuration of objects
- Considers:
  - Data in macroscopic scale
  - Object geometries
  - Relation between objects and prior knowledge
- **Problem:**
  - **MPP is limited to low-parametric geometries (disks/ellipses/tubes/lines)**



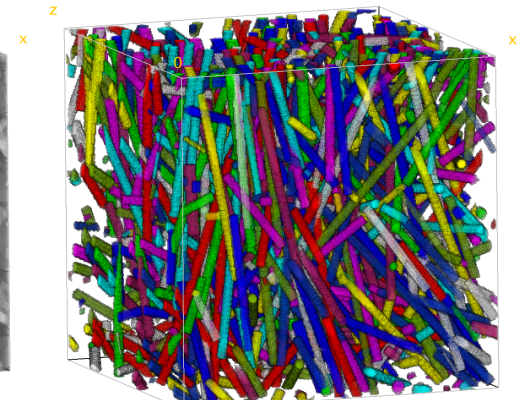
NiAlCr



Ellipse Model  
[Zhao et al. 2016]



Fiber Reinforced Composite  
[ACME Lab Purdue]



Connected Tube  
[Li et al. 2018]

# Illustrative dataset

## Objective:

Characterization of glass fiber reinforced composite:

- **Structural Features**

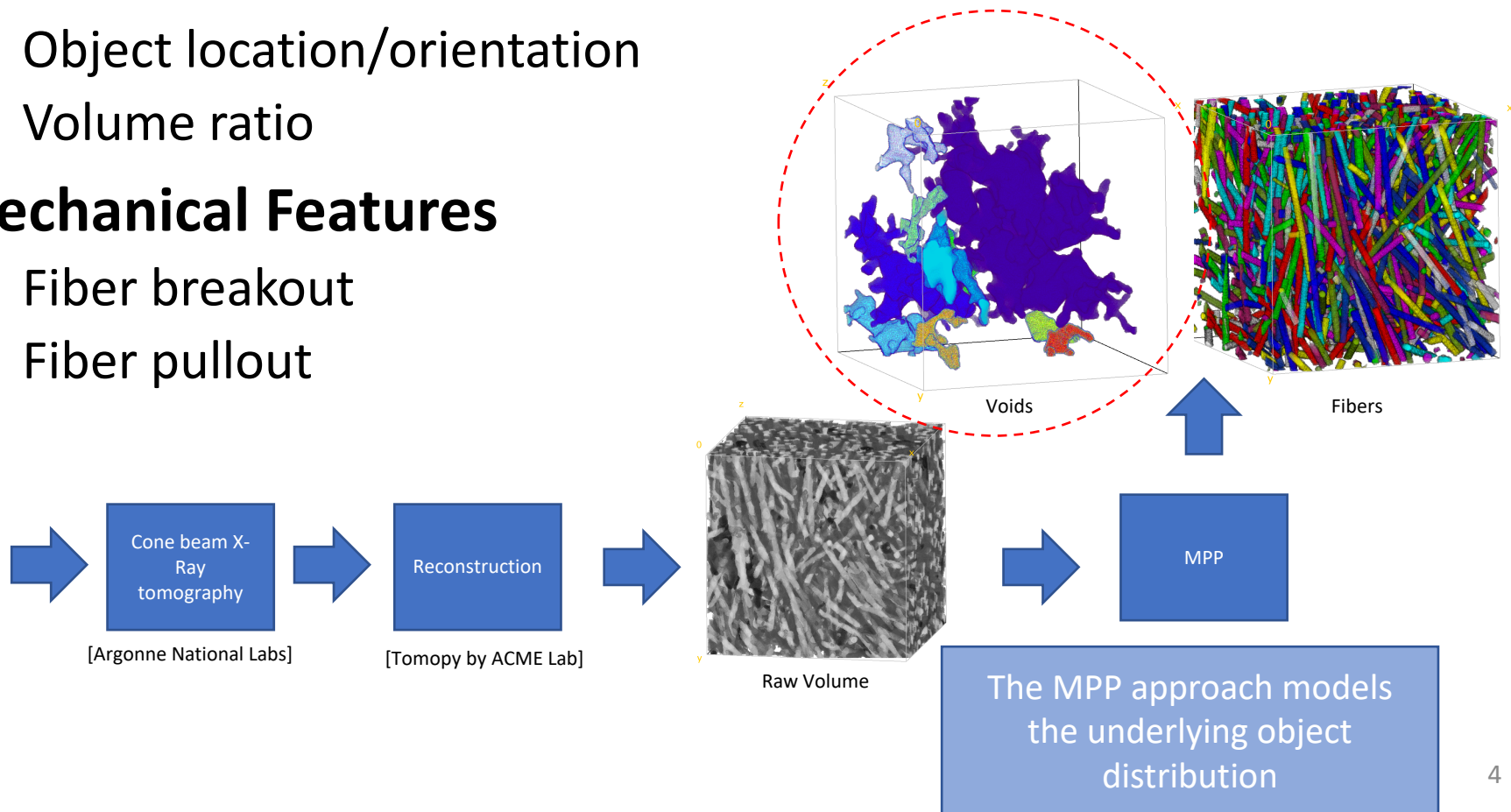
- Object location/orientation
- Volume ratio

- **Mechanical Features**

- Fiber breakout
- Fiber pullout



[Dupont]



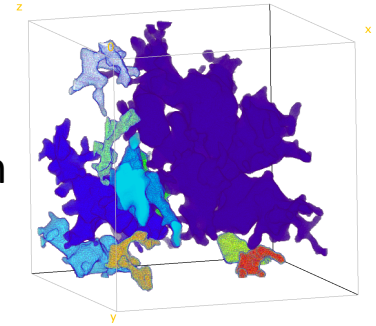
# Challenges

- Irregular shapes

- Active Contour Modeling



Void representation  
in composite:

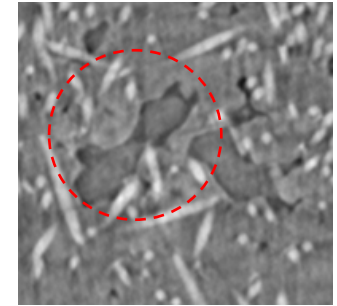


- Low contrast

- Balloon method (MPP-AC)
- Hybrid LS method (MPP-LS)



Composite cross  
section:



[ACME Lab Purdue]

- Large datasets

- Hybrid LS method



Volume Size:

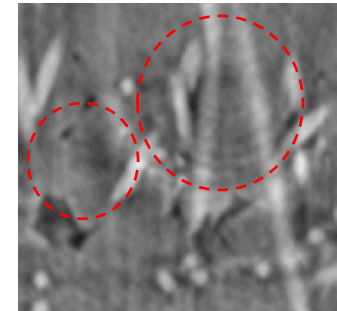
2500 × 2500 × 1300  
voxels

- Imaging and reconstruction noise

- 3D Filtering



Composite cross  
section:



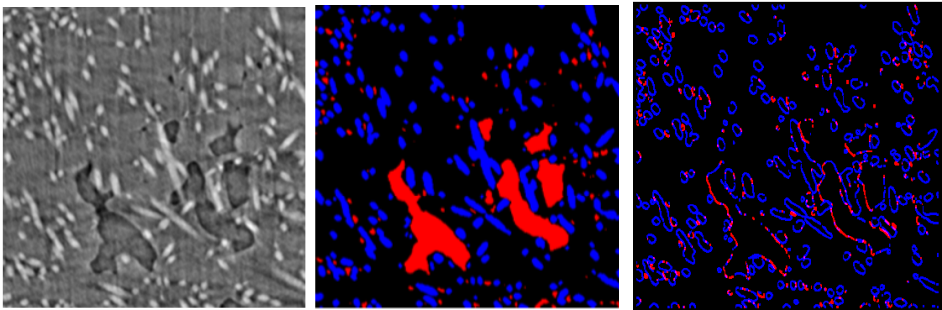
[ACME Lab Purdue]

# Common Segmentation Approaches

## Machine Learning

Discriminative Dictionary Learning: Edge Classifier

Sparse training data



Original Image

Labeled Image

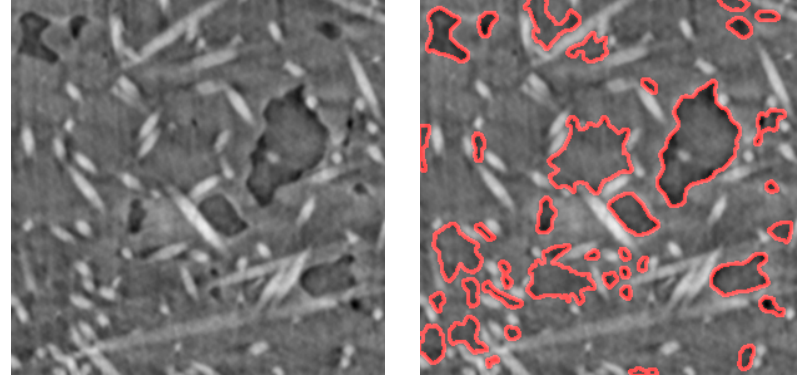
Classified Edges

[Mairal 2008]

## Active Contours only

Hybrid Level Sets

Initialization dependent



Original Image

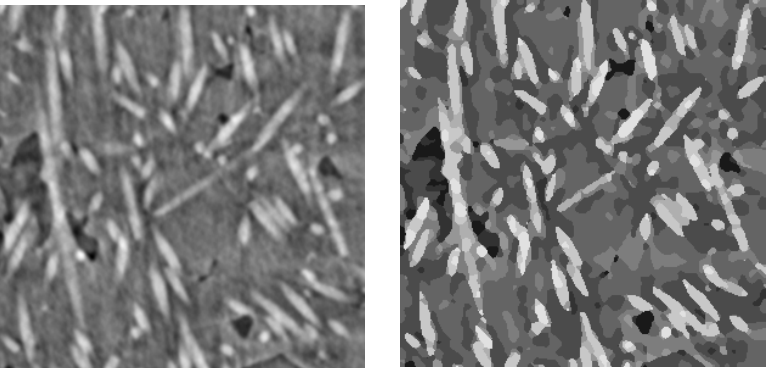
Detected Contours

[Yan 2008]

## Markov Random Fields

EM/MPM

Pixelwise segmentation



Original Image

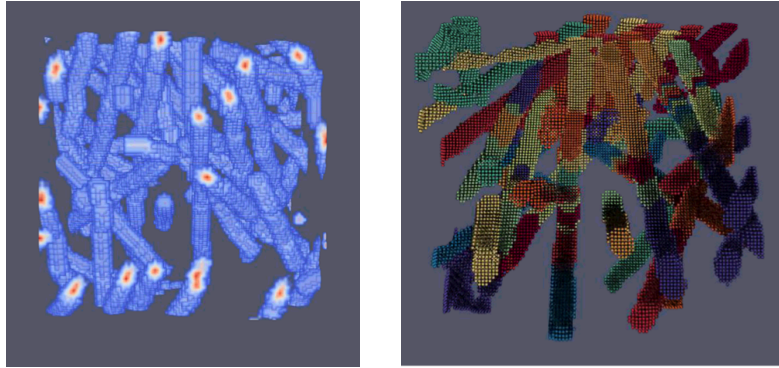
EMMPM. Labels = 9

[Comer 2000]

## Watershed

Watershed by flooding

Requires careful energy/marker setting



Distance Transform

Watershed Segmentation

[Beucher 1979]

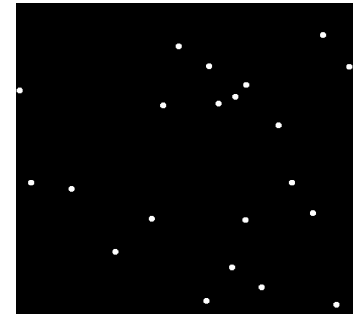
# Marked Point Process



# Marked Point Process

- Set up point process

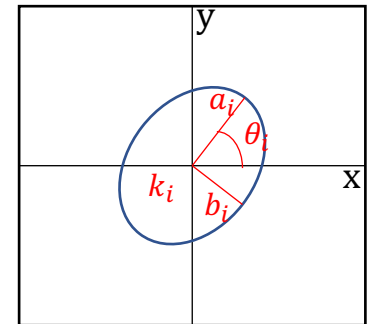
- Define a Point Process  $\mathbf{x}$  on lattice  $K \subset \mathbb{R}^d$
- Each point  $k_i$  in  $\mathbf{x} = \{k_1, \dots, k_n\}$  denotes a coordinate.
- $n$  is a random variable



Realization of a Point Process

- Set up marks

- A mark space  $M$  describes objects' parameters
- Single marked object is  $\omega_i = (k_i, m_i) \in K \times M$



Sample marked object  
 $\omega_i = (k_i, a_i, b_i, \theta_i)$

- MPP = point process + marks

- An object configuration is  $\mathbf{w} = \{\omega_1, \dots, \omega_n\}$
- An MPP  $\mathbf{w}$  is defined on space  $\Omega = K \times M$



Realization of an MPP

# MPP density

Density:

$$p(\mathbf{w}) = \frac{1}{Z_{\Omega}} \exp(-U(\mathbf{w}))$$

$\mathbf{w}$ : Marked object configuration

$\omega_i$ : Single Marked  $i^{th}$  object

$Z_{\Omega}$ : Normalizing constant

$\omega_i \sim \omega_j$ : Neighbor relation

Gibbs Energy:

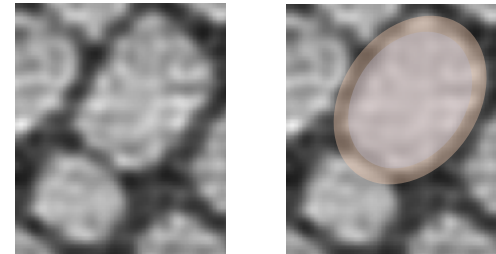
$$U(\mathbf{w}) = \sum_{\omega_i \in \mathbf{w}} U_d(\omega_i) + \sum_{\substack{\omega_i \sim \omega_j \\ \omega_i, \omega_j \in \mathbf{w}}} U_p(\omega_i, \omega_j)$$

data term                      prior term

Data Term:

$$U_d(\omega_i) \propto \text{object fitting}$$

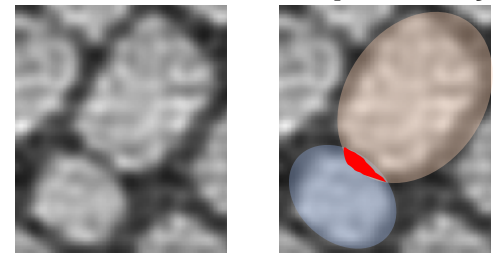
Data Term:  $U_d(\omega_i)$



Prior Term:

$$U_p(\omega_i, \omega_j) \propto \text{overlap penalizer}$$

Prior Term:  $U_p(\omega_i, \omega_j)$



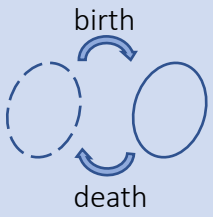
# Energy Optimization

Goal: find optimal configuration

$$\hat{\mathbf{w}} = \arg \max_{\mathbf{w} \in \Omega} p(\mathbf{w}) = \arg \min_{\mathbf{w} \in \Omega} U(\mathbf{w})$$

Use Markov Chain Monte Carlo with stochastic annealing

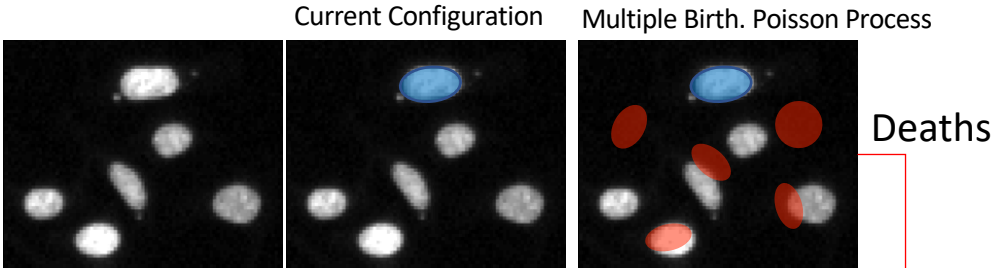
- Markov Chain on  $\Omega$  needs to be:
- Finite
  - Aperiodic
  - Irreducible
  - Reversible



Annealing scheme:

$$T^{k+1} = T^k \alpha, \quad \alpha \in (0, 1)$$

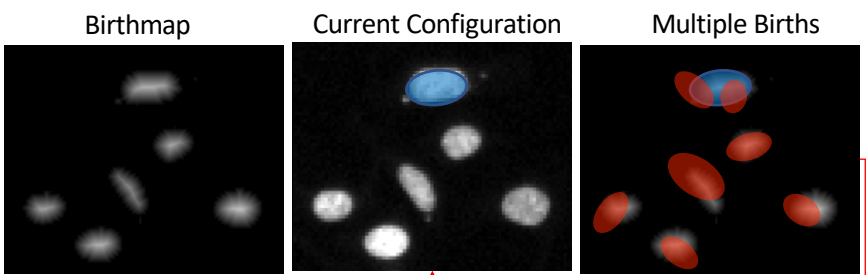
## Multiple Birth and Death (1)



$$\mathbf{w} = \{\omega_1\} \quad \mathbf{w}' = \{\omega'_1, \omega'_2 \dots \omega'_n\}$$

[Descombes 09]

## Multiple Birth and Death (2)



Deaths 10

$$\mathbf{w} = \{\omega_1\} \quad \mathbf{w}' = \{\omega'_1, \omega'_2 \dots \omega'_n\}$$

[Kaggle Datascience Bowl 2018]

# MPP-Active Contours

# Active Contour Model

- Define curve:  $C_t = \{(x_t, y_t)\}$ , where  $t \in [0, 2\pi]$

- Energy Function:

$$E(C_t) = \int_0^{2\pi} E_{\text{int}}(C_t) + E_{\text{ext}}(C_t) dt$$

- Internal Energy:

$$E_{\text{int}}(C_t) = \int_0^{2\pi} \frac{1}{2} [\alpha |C_t'|^2 + \beta |C_t''|^2] dt$$

Elastic Term Curvature Term

- External Energy:

$$E_{\text{ext}}(C_t) = -\kappa_1 |\nabla I(x_t, y_t)|^2 - \kappa_2 |I_{\text{dark}}(x_t, y_t)| + \kappa_3 \vec{n}_C(t)$$

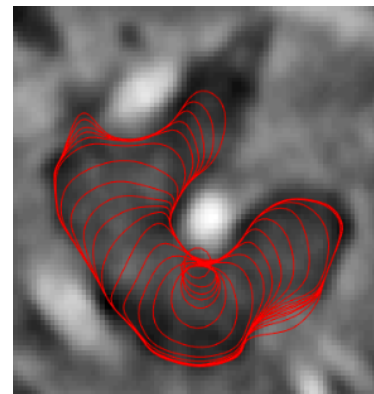
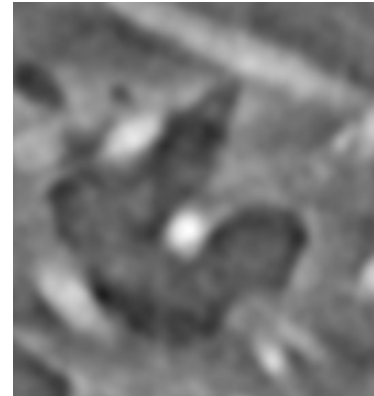
Stop at edges

Attract to dark regions

Inflate the Contour

[Cohen 1993]

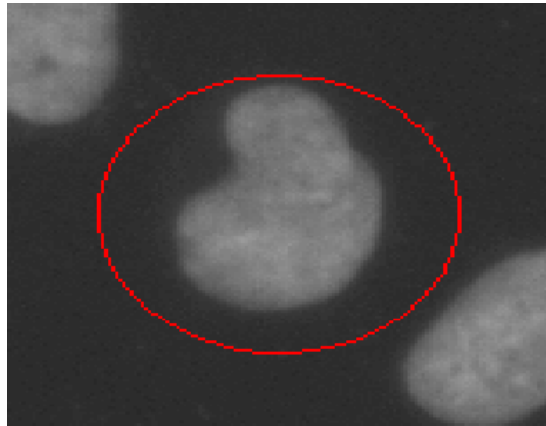
$C_t$ : Parametric contour  
 $C_t'$ : First derivative w/r to  $t$   
 $C_t''$ : Second derivative w/r to  $t$   
 $x_t, y_t$ : Coordinates in contour  
 $\vec{n}_C$ : Normal to the contour  
 $I$ : Image Domain  
 $I_{\text{dark}}$ : Image is 1 in pixels with low intensities



[Kass 1988]

# Boundary Parameters

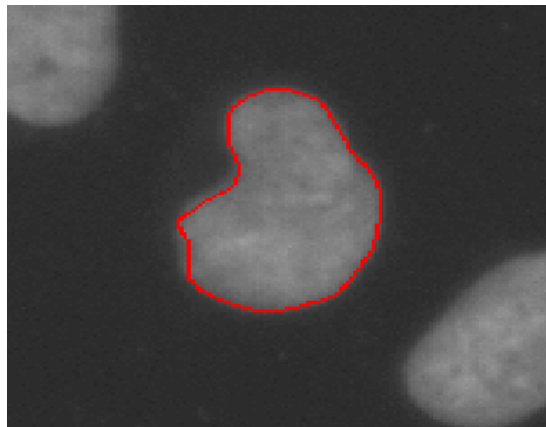
$$E(C_t) = \int_0^{2\pi} \frac{1}{2} [ |C'_t|^2 + \beta |C''_t|^2 - 0.05 |\nabla I(x_c, y_c)|^2 - 0.1 \vec{n}_c(t) ] dt$$



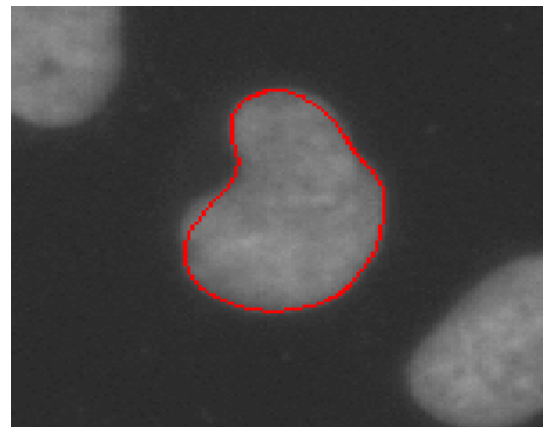
Initial Contour



$\beta = 0$



$\beta = 10$



$\beta = 100$

Boundary Energies

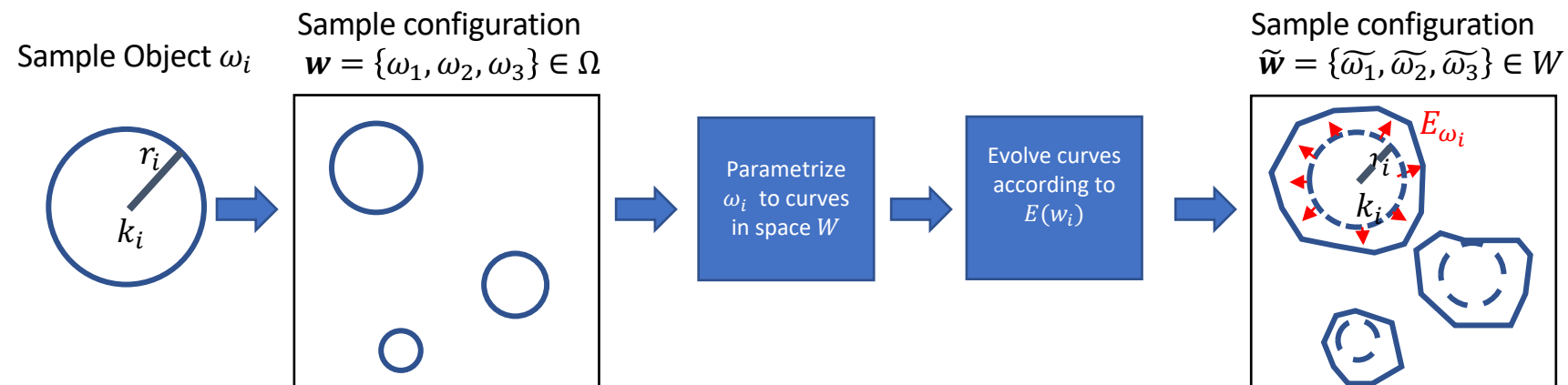
$\beta$	$C'(t)$	$C''(t)$
0	876.46	44.83
10	818.88	15.77
100	746.83	5.00

# Active Contours in the MPP

$K$ : Image lattice  
 $\Omega$ : Configuration space  
 $W$ : Parametric space  
 $\mathbf{w}$ : Marked object configuration  
 $\omega_i$ : Single marked  $i^{\text{th}}$  object  
 $\tilde{\omega}_i$ : Evolved marked  $i^{\text{th}}$  object  
 $\tilde{\mathbf{w}}$ : Evolved object configuration

- Initial Mark Object Field:
  - Disks with mark  $\omega_i = \{k_i, m_i\} \in \Omega, \Omega \subset K \times M$
  - $M = [r_{\min}, r_{\max}]$
- Modified Marked Object Field:
  - Define energy functional  $E(\omega_i)$  on space  $W$
  - Parametrize curve  $\omega_t \in W$
  - Perform energy minimization on  $E(\omega_t)$  to evolve  $\omega_t$  into  $\tilde{\omega}_t \in W$

[Kulikova, 2009]



# Energies

$\mathbf{w}$ : Marked object configuration  
 $\omega_i$ : Single marked  $i^{th}$  object  
 $\tilde{\omega}_i$ : Evolved marked  $i^{th}$  object  
 $\tilde{\mathbf{w}}$ : Evolved object configuration  
 $z$ : Normalizing Constant  
 $\omega_i \sim \omega_j$ : Neighbor Relation

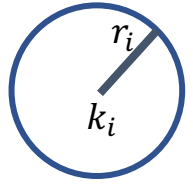
## MPP-AC Energy

- Gibbs Process with probability density

$$p(\mathbf{w}) = \frac{1}{Z} \exp\{-U(\mathbf{w})\}$$

- Energy Function

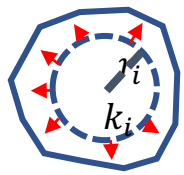
$$U(\mathbf{w}) = \sum_{\omega_i \in \mathbf{w}} U_d(\omega_i) + \sum_{\omega_i \sim \omega_j} U_p(\omega_i, \omega_j)$$



$\omega_i$

- Data Energy: Active Contour Energy

$$U_d(\omega) = \min_{\omega_i} \left\{ \int_0^1 E_{\text{int}}(\omega_t) + E_{\text{ext}}(\omega_t) dt \right\} = U_d(\tilde{\omega}_t)$$



$\tilde{\omega}_i$

- Prior Energy

$$U_p(\omega_i, \omega_j) = \begin{cases} A_{\text{overlap}}(\tilde{\omega}_{t_i}, \tilde{\omega}_{t_j}) & \text{if } A_{\text{overlap}}(\tilde{\omega}_{t_i}, \tilde{\omega}_{t_j}) \leq T_{\text{overlap}} \\ \infty & \text{otherwise} \end{cases}$$



# Simulation: Multiple Birth and Death

$b_o$ : Birth Rate  $\in [0,1]$   
 $T$ : Process Temperature  
 $\sigma$ : Process Intensity

## Algorithm 1 Multiple Birth and Death Algorithm

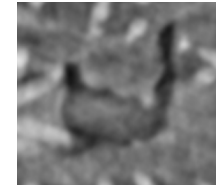
```

1: procedure MPP ENERGY MINIMIZATION
2:   Initialization:
3:   Create birthmap  $b_o$ 
4:   Initialize  $b_{rate} = b_o, T = T_o, \sigma = \sigma_o.$ 
5:   Birth Step:
6:     Visit pixels in raster order
7:      $\omega' \leftarrow$  draw a sample from space  $\Omega$ 
8:     Add  $\omega'$  to configuration  $\mathbf{w}$  with probability  $\sigma b_{rate}$ 
9:     Evolve  $\omega'$  to  $\tilde{\omega}'$ 
10:  Death Step:
11:  Sort all elements of  $\mathbf{w}$  by decreasing energy.
12:  For every object  $\omega_i$  in  $\mathbf{w}$  calculate:
13:  
$$d_{rate}(\omega_i) = \frac{\sigma^{(k)} \exp \frac{U(\mathbf{W}|\mathcal{Y}) - U(\mathbf{W} - \omega_i|\mathcal{Y})}{T^k}}{1 + \sigma^{(k)} \exp \frac{U(\mathbf{W}|\mathcal{Y}) - U(\mathbf{W} - \omega_i|\mathcal{Y})}{T^k}}$$

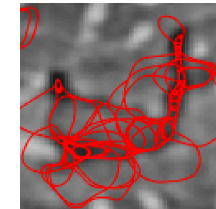
14:  Delete  $\omega_i$  with probability  $d_{rate}(\omega_i)$ 
15:  Convergence Test:
16:  if all the elements born during the birth step are killed
    during the death step
17:    terminate process
18:  else
19:     $T^{k+1} \leftarrow T^k \times \alpha, \sigma^{k+1} \leftarrow \sigma^k \times \alpha \alpha \in (0,1)$ 
20:    goto Birth Step
21:
22: end procedure
  
```



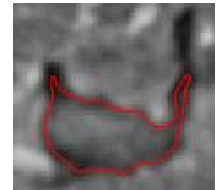
**Initialization**  
(birth map)



**Birth Step**



**Death Step**



Multiple Birth and Death Algorithm

# Simulation: Multiple Birth and Death

$b_o$ : Birth Rate  $\in [0,1]$   
 $T$ : Process Temperature  
 $\sigma$ : Process Intensity

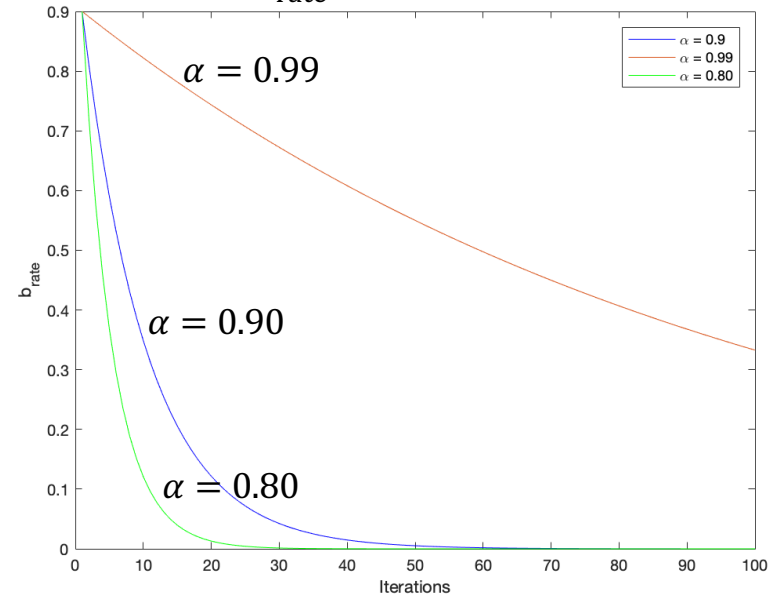
## Algorithm 1 Multiple Birth and Death Algorithm

- 1: **procedure** MPP ENERGY MINIMIZATION
- 2:     **Initialization:**
- 3:     Create birthmap  $b_o$
- 4:     Initialize  $b_{rate} = b_o, T = T_o, \sigma = \sigma_o$ .
- 5:     **Birth Step:**
- 6:         Visit pixels in raster order
- 7:          $\omega' \leftarrow$  draw a sample from space  $\Omega$
- 8:         Add  $\omega'$  to configuration  $\mathbf{w}$  with probability  $\sigma b_{rate}$
- 9:         Evolve  $\omega'$  to  $\tilde{\omega}'$
- 10:     **Death Step:**
- 11:     Sort all elements of  $\mathbf{w}$  by decreasing energy.
- 12:     For every object  $\omega_i$  in  $\mathbf{w}$  calculate:
 
$$d_{rate}(\omega_i) = \frac{\sigma^{(k)} \exp \frac{U(\mathbf{W}|\mathcal{Y}) - U(\mathbf{W} - \omega_i|\mathcal{Y})}{T^k}}{1 + \sigma^{(k)} \exp \frac{U(\mathbf{W}|\mathcal{Y}) - U(\mathbf{W} - \omega_i|\mathcal{Y})}{T^k}}$$
- 13:     Delete  $\omega_i$  with probability  $d_{rate}(\omega_i)$
- 14:     Delete  $\omega_i$  with probability  $d_{rate}(\omega_i)$
- 15:     **Convergence Test:**
- 16:     **if** all the elements born during the birth step are killed during the death step
- 17:         terminate process
- 18:     **else**
- 19:          $T^{k+1} \leftarrow T^k \times \alpha, \sigma^{k+1} \leftarrow \sigma^k \times \alpha \alpha \in (0,1)$
- 20:         **goto** Birth Step
- 21:
- 22: **end procedure**

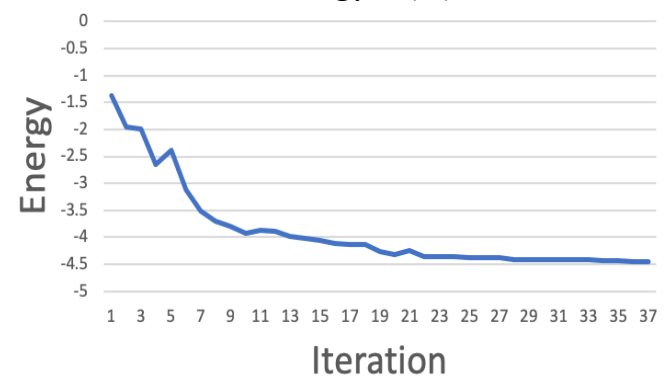
$\sigma$ : Process Intensity

Energy Change  
&  
Temperature dependent

$\sigma b_{rate}$  vs Iterations



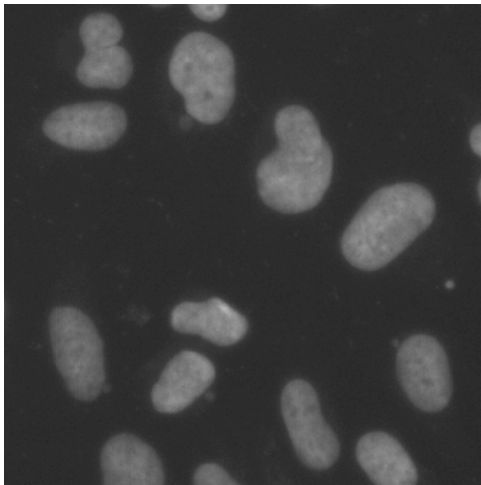
Unnormalized Energy  $U(\mathbf{w})$ .  $\alpha = 0.90$



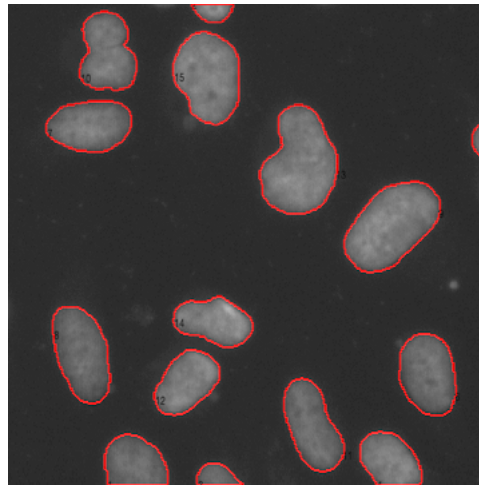
# Sample Results I: Human Cells

$$E(C_t) = \int_0^{2\pi} \frac{1}{2} [ |C_t'|^2 + \beta |C_t''|^2 - 0.05 |\nabla I(x_c, y_c)|^2 ] dt$$

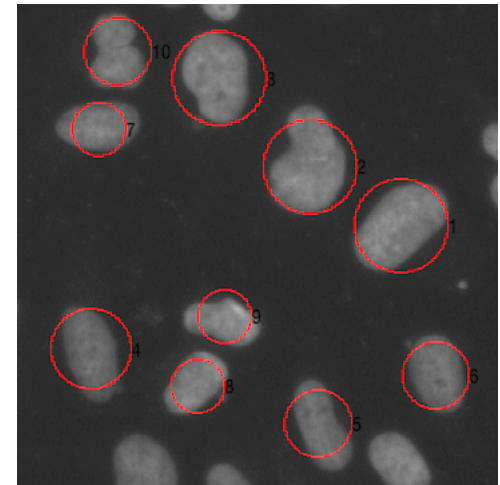
[Kaggle Datascience Bowl 2018]



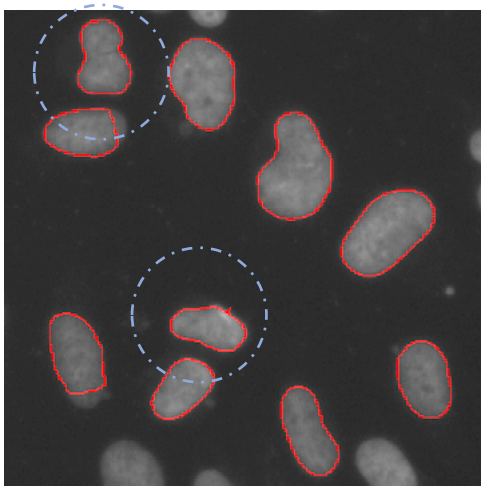
Original Image



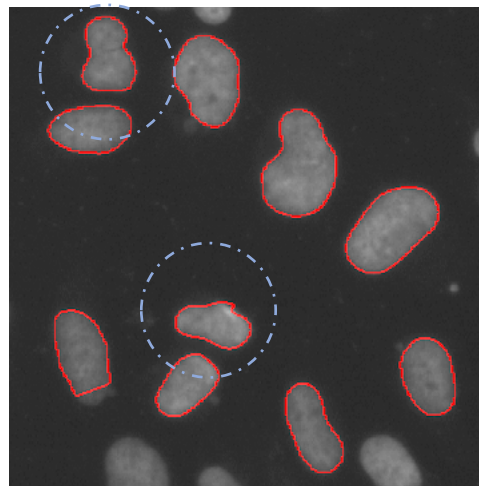
Ground Truth



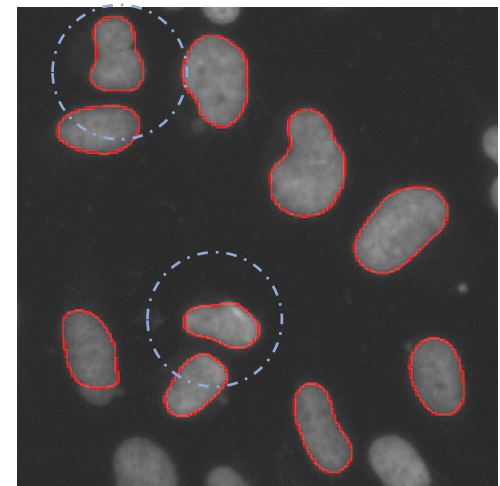
Marks



$\beta = 1$

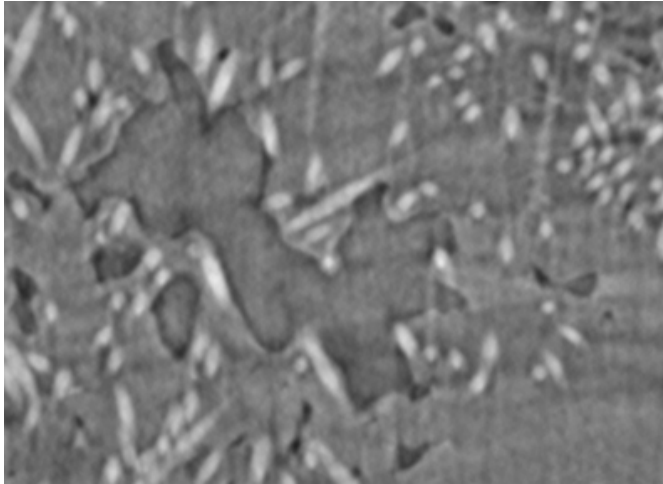


$\beta = 10$

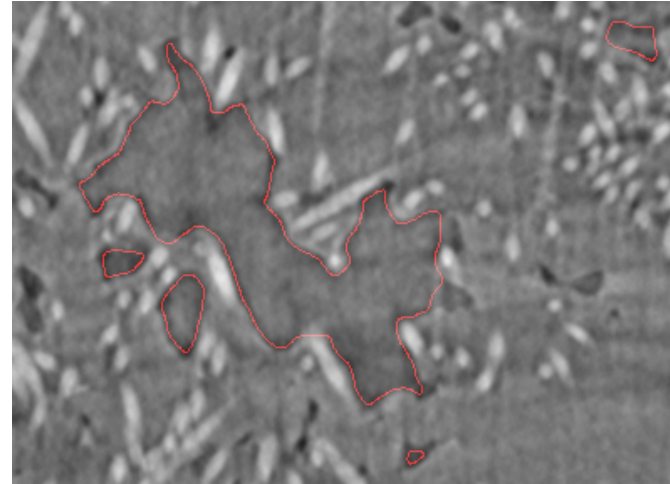


$\beta = 100$

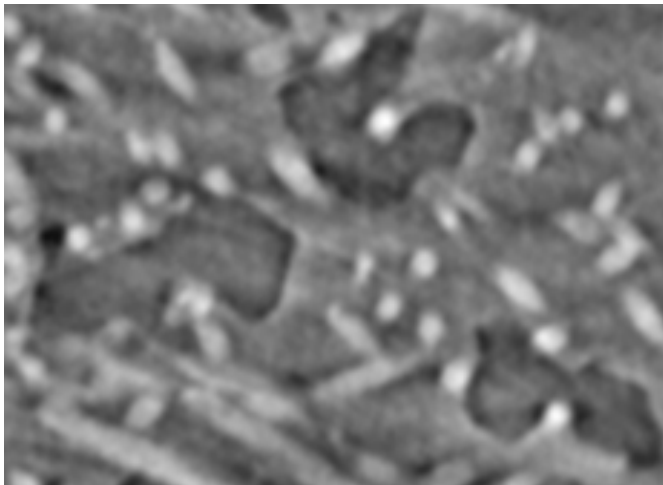
# MPP-AC Results II: voids



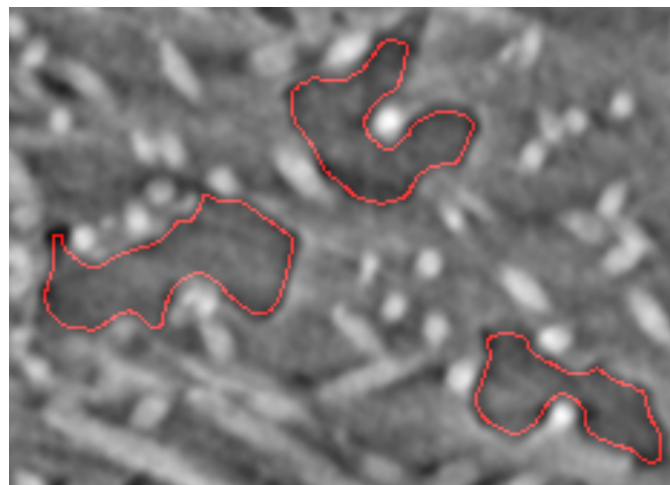
Original Image



MPP-AC

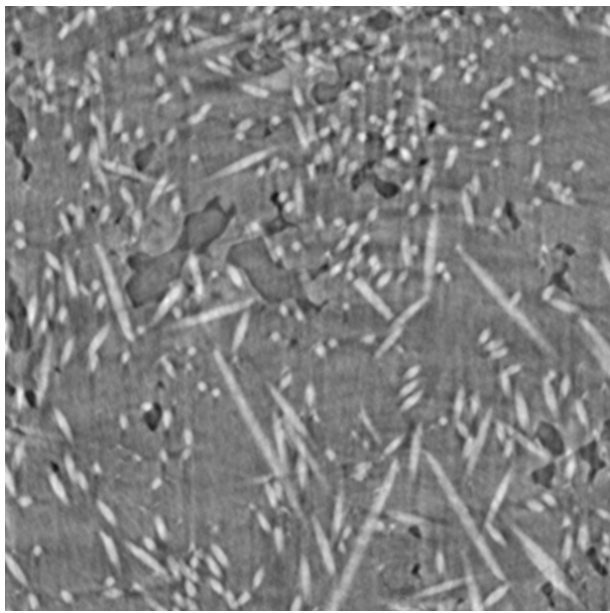


Original Image

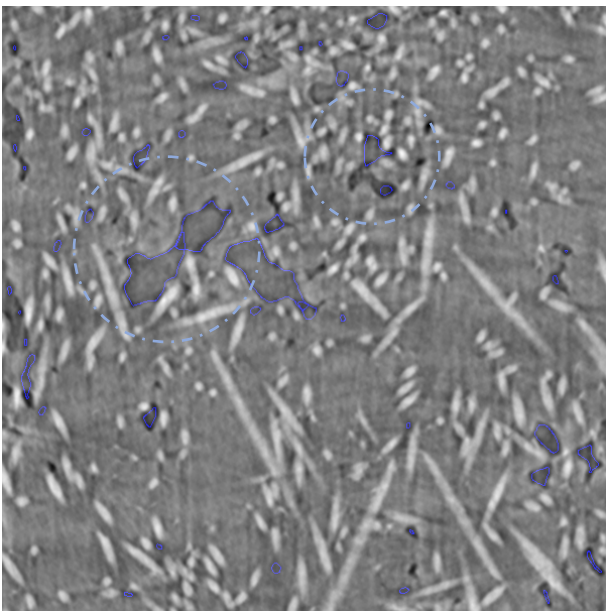


MPP-AC

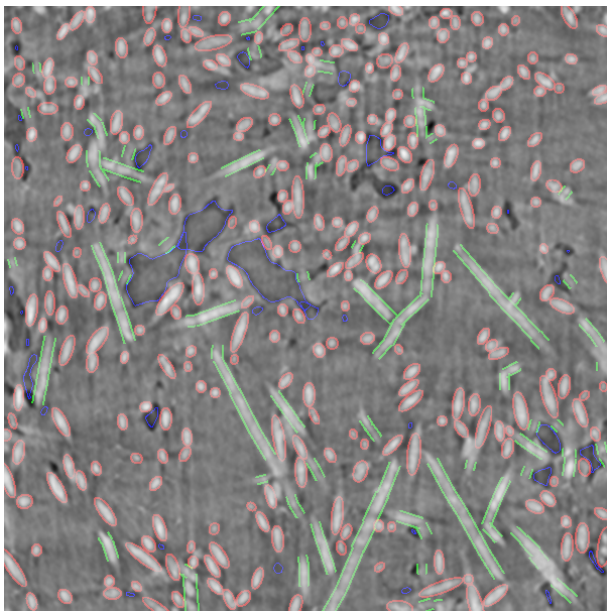
# MPP-AC Results III: voids and fibers



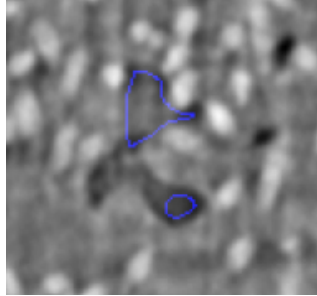
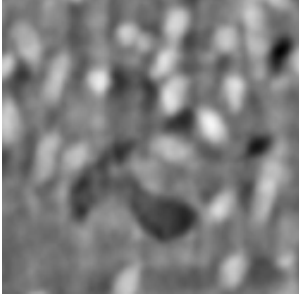
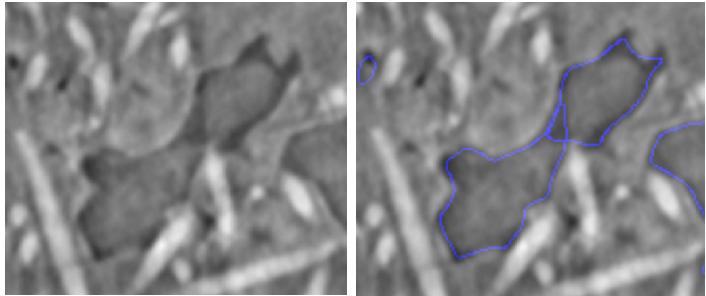
Original Image



MPP-AC for voids only



MPP-AC combined with MPP Connected tube



Parametrization Constrains

# Contributions of this work

---

- Exploration of the MPP-AC to microscopy images
- Adaptation of the classic AC energy that involves smoothness and curvature.
  - Exploration of the curvature weighting effects
- Adaptation of the balloon force to capture objects with irregular shapes

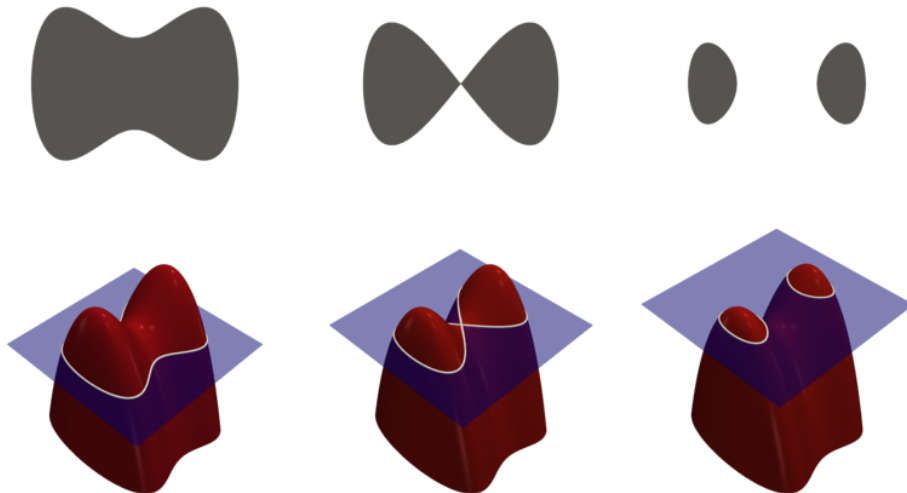
# MPP-Level Sets

# What is a level set?

Given a function  $\phi: \mathbb{R}^d \rightarrow \mathbb{R}$

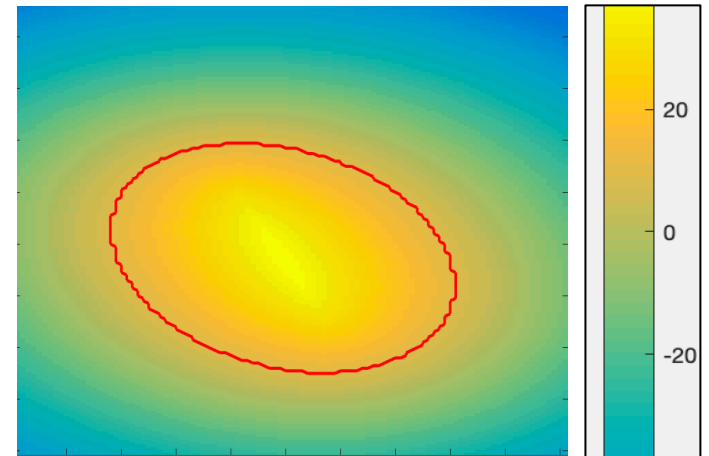
Curve:  $C_t = \{k \in \mathbb{R}^d \mid \phi(k) = 0\}$  is the zeroth level set of  $\phi$

Example of level sets and object representations



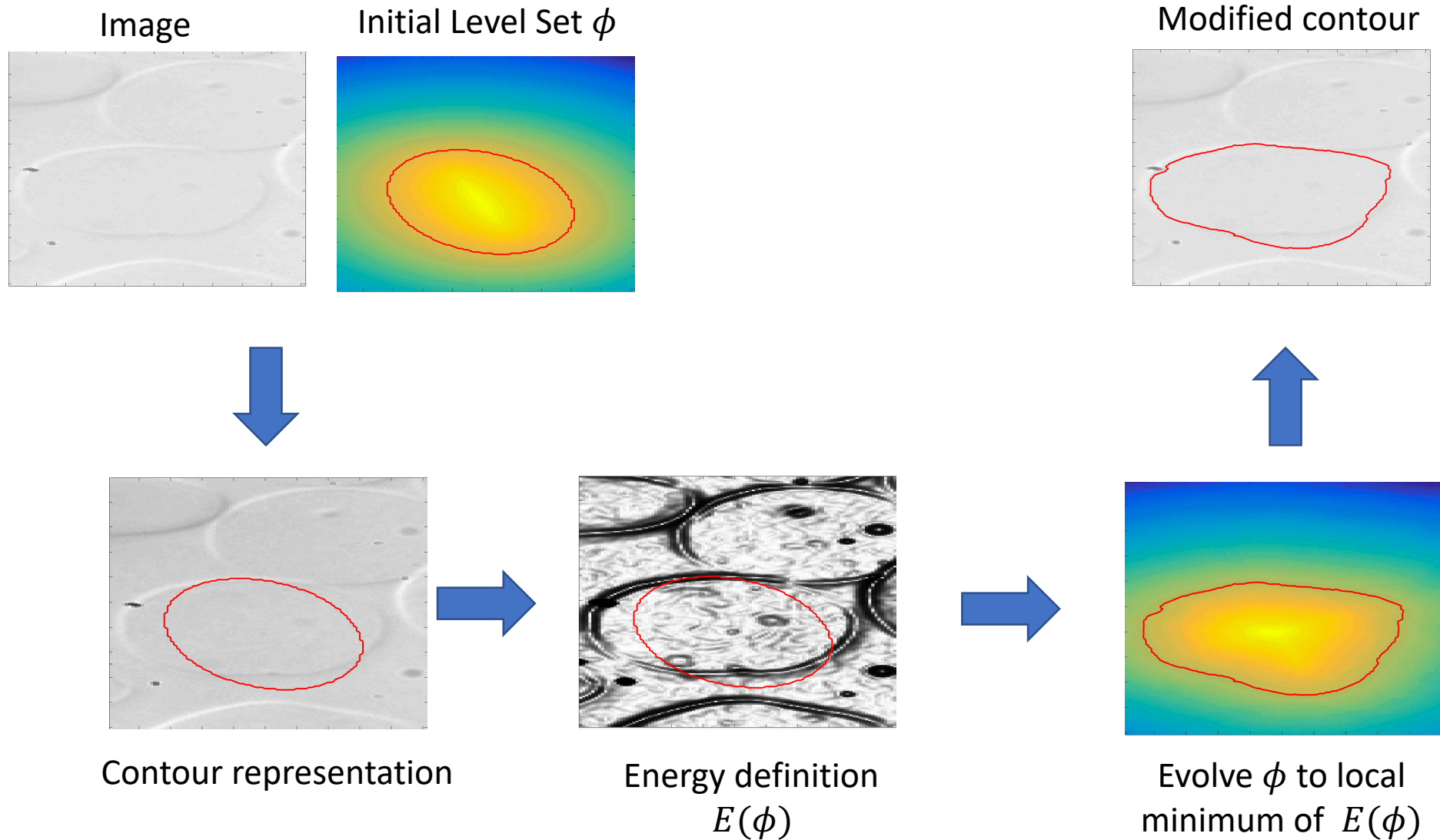
[Wikipedia]

Example of initial level set of  $\phi$



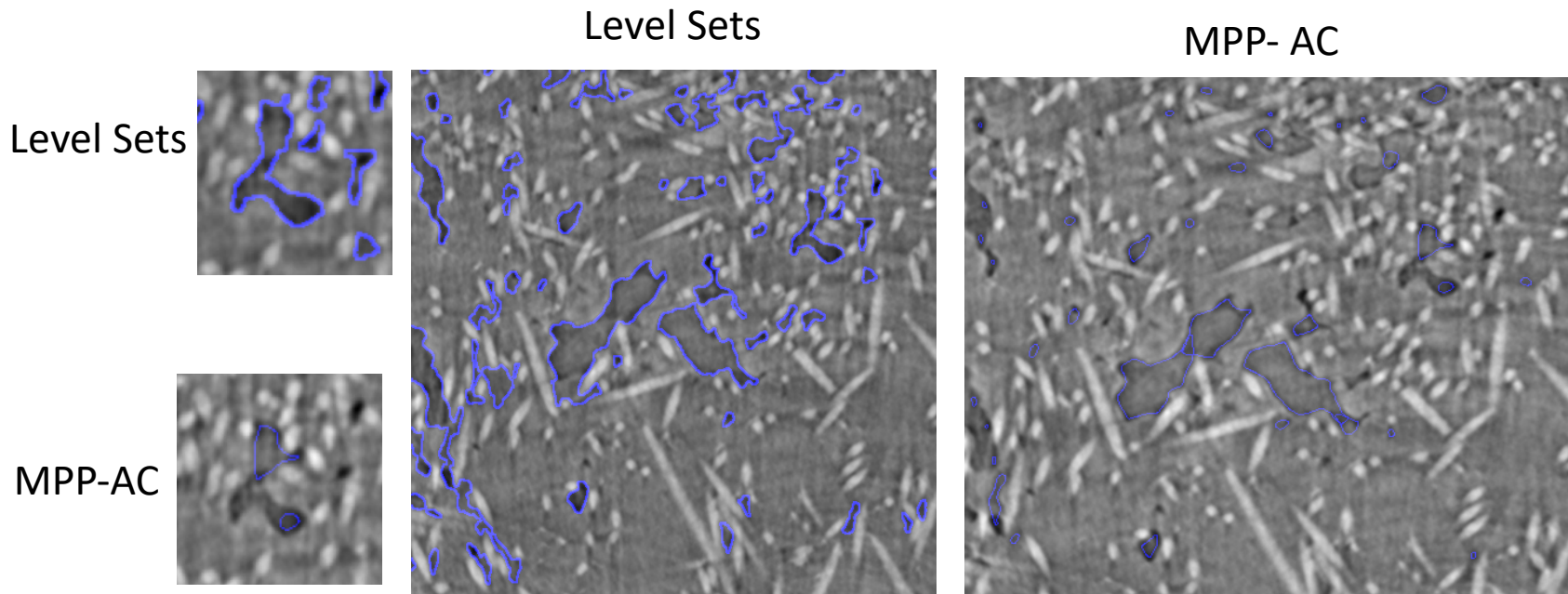


# Summary of level sets approach



# Advantages of LS vs parametric AC

- Level Sets can:
  - Adapt better to topological changes
  - Allow object merging and splitting
  - Facilitate the dimension increase



# Hybrid Level Sets Energy + Shape prior

- Energy Function:

$$E(\phi) = \alpha E_{\text{region}}(\phi) + \beta E_{\text{edge}}(\phi) + \gamma E_{\text{shape}}(\phi)$$

[Yan 2008]

$K$ : Image domain  
 $\phi$ : Embedding function  
 $H(\cdot)$ : Heaviside function  
 $g(\cdot)$ : Edge function  
 $\phi_o$ : Level set geometric prior

$$E_{\text{region}}(\phi) = \int_{k \in K} (k - \mu) H(\phi) dk$$

Attract to dark regions

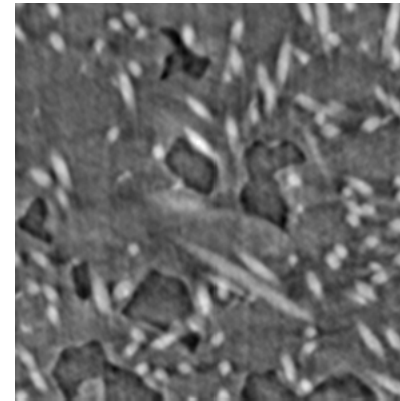
$$E_{\text{edge}}(\phi) = \int_{k \in K} g(k) |\nabla H(\phi)| dk$$

Attract to edges

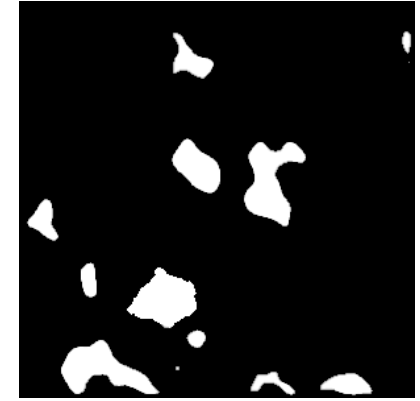
$$E_{\text{prior}}(\phi) = \int_{k \in K} (H(\phi) - H(\phi_o))^2 dk$$

Preserve shape

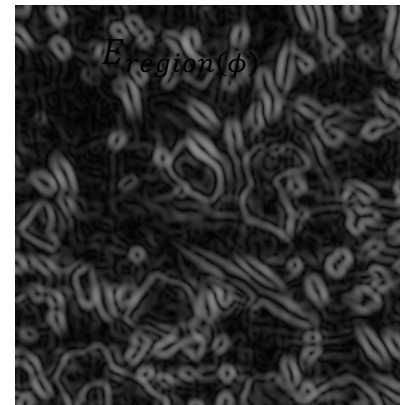
Preserve irreducible Markov Chain



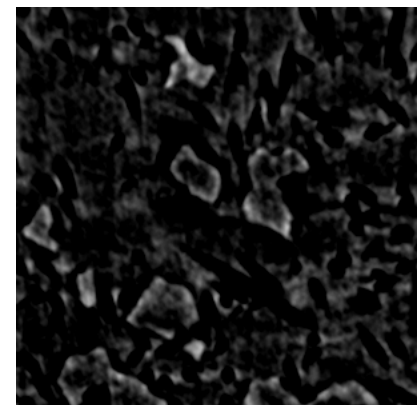
Original Image



Manual label



$E_{\text{edge}}$



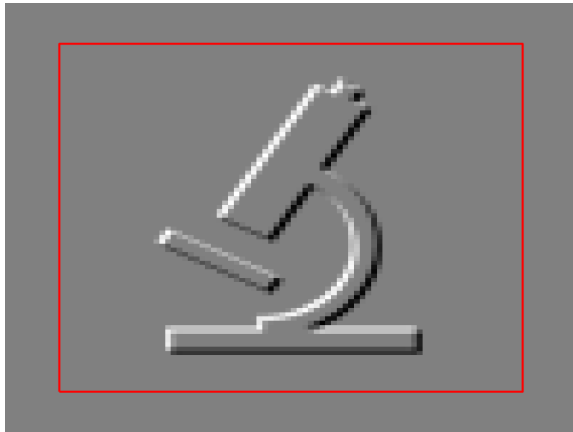
$E_{\text{region}}$

# Hybrid Level Sets Boundary Penalizer

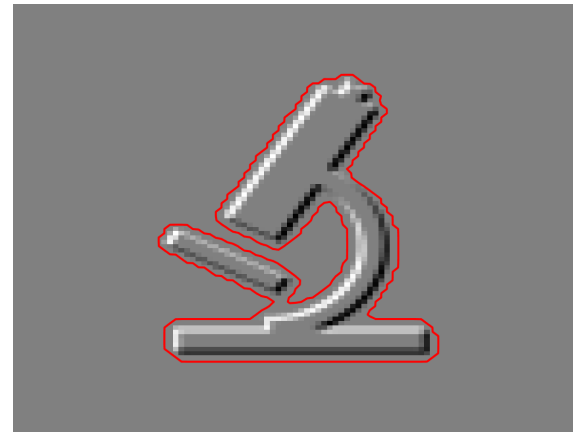
$$E(\phi) = \int_{k \in K} g(k) |\nabla H(\phi)| dk$$

$$\phi_t = \beta \operatorname{div}(g(k) \nabla \phi)$$

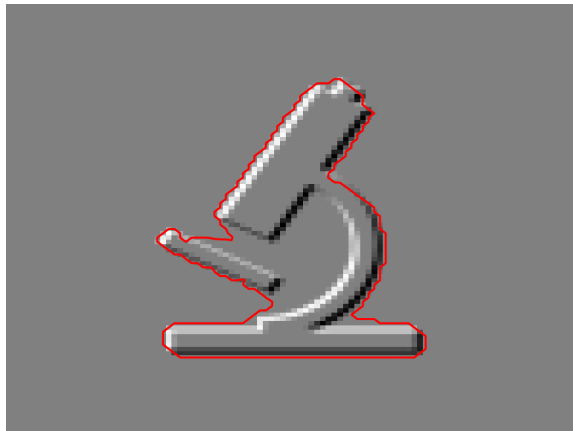
$$g(k) = \frac{1}{1+c |\nabla f_\sigma * K|}$$



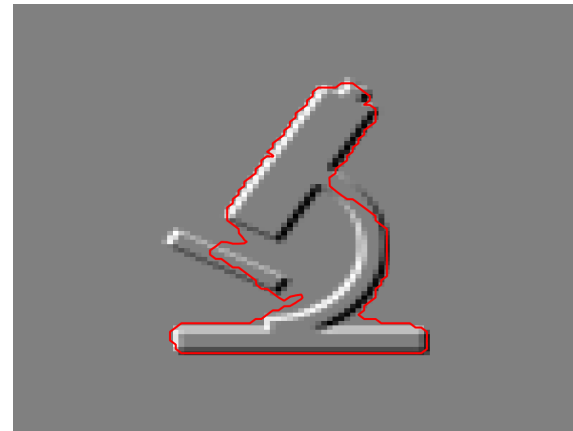
Initialization



$\beta = 1$



$\beta = 10$



$\beta = 20$

**div**: divergence operator  
**c**: Slope constant  
 **$f_\sigma$** : Gaussian filter  
**\***: Convolution operator

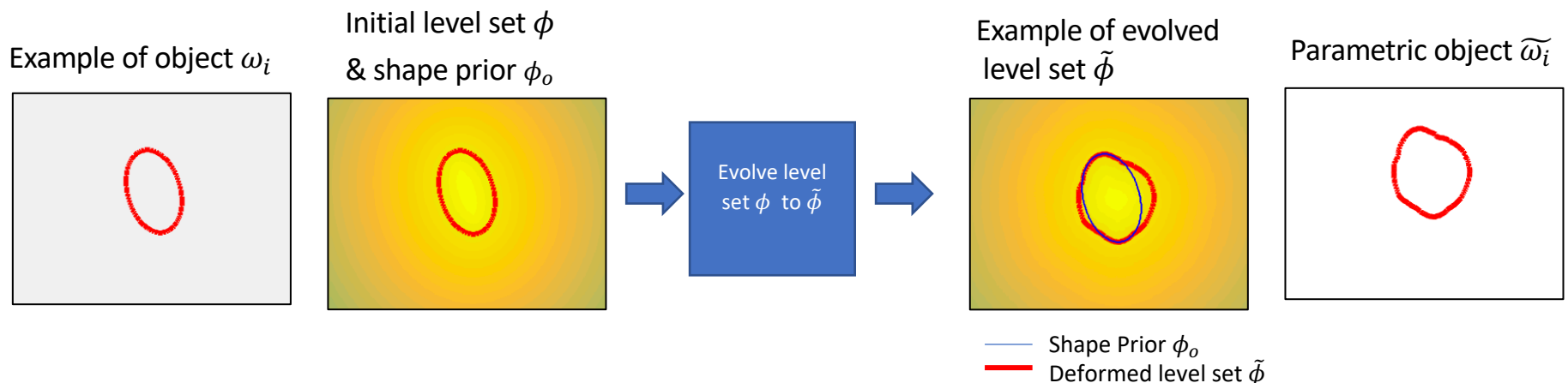
# Level Sets in the MPP

- MPP Object Field:

- Ellipses with mark  $\omega_i = (k_i, m_i) \in \Omega$
- $M = [a_{\min}, a_{\max}] \times [b_{\min}, b_{\max}] \times [\theta_{\min}, \theta_{\max}]$

- Marked Object:

- Use MPP object  $\omega = (k_i, m_i)$  as initialization and shape prior  $\phi_o$
- Evolve level set  $\phi$  to  $\tilde{\phi}$
- Parametrize evolved level set  $\tilde{\phi}$  to  $\tilde{\omega}(t)$



# From LS to parametric energy

Level set energy:

$$E(\phi) = \int_{k \in K} \alpha(k - \mu)H(\phi) + \beta g_\sigma(k)|\nabla H(\phi)| + \gamma(H(\phi) - H(\phi_o))^2 dk$$

Parametric energy: ↓ regions ↓ edges

$$E(\omega_i) = \frac{1}{|D\tilde{\omega}_i|} \int_{t \in D\tilde{\omega}_i} \alpha(t - \mu)dA + \frac{1}{|d\tilde{\omega}_i|} \int_{t \in d\tilde{\omega}} \beta g_\sigma(t)dt$$

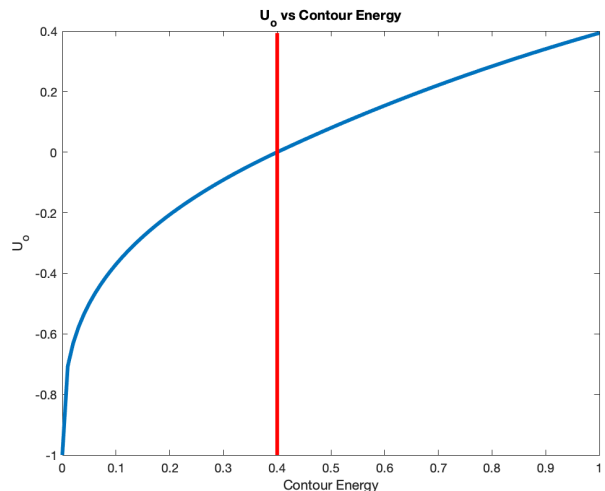
$D\tilde{\omega}$ : Area inside  $\tilde{\omega}_i$

$d\tilde{\omega}$ : Line denoting contour  $\tilde{\omega}_i$

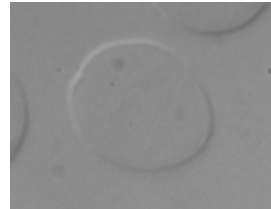
Quality Term:

$$U_d(\omega_i) = \begin{cases} 1 - \exp\left(-\frac{E(\hat{\omega}) - E_o}{3E_o}\right) & E(\hat{\omega}) \geq E_o \\ \left(\frac{E(\hat{\omega})}{E_o}\right)^{1/3} - 1 & \text{otherwise} \end{cases}$$

$E_o = 0.40$



Original Image

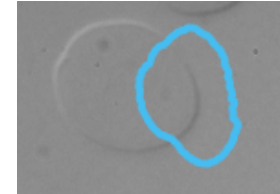


Good fit



$E(\omega) = 0.32$   
 $U_d(\omega) = -0.06$   
 $d_{rate}(\omega) = 0.33$

Bad fit



$E(\omega) = 0.76$   
 $U_d(\omega) = 0.26$   
 $d_{rate}(\omega) = 0.92$

# MPP-LS Energy

$\mathbf{w}$ : Marked Object Configuration  
 $\omega_i$ : Single Marked  $i^{th}$  Object  
 $\tilde{\omega}_i$ : Evolved Marked  $i^{th}$  Object  
 $z$ : Normalizing Constant  
 $\omega_i \sim \omega_j$ : Neighbor Relation  
 $E_o$ : Contour energy parameter

- Gibbs Process with probability density

$$p(\mathbf{w}) = \frac{1}{Z} \exp\{-U(\mathbf{w})\}$$

- Energy Function

$$U(\mathbf{w}) = \sum_{\omega_i \in \mathbf{W}} U_d(\omega_i) + \sum_{\omega_i \sim \omega_j} U_p(\omega_i, \omega_j)$$

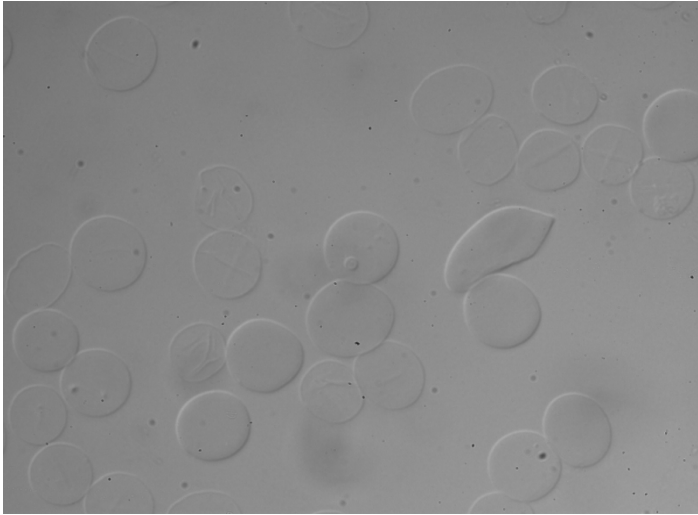
- Prior Energy

$$U_p(\omega_i, \omega_j) = \begin{cases} A_{\text{overlap}}(\tilde{\omega}_i, \tilde{\omega}_j) & \text{if } A_{\text{overlap}}(\tilde{\omega}_i, \tilde{\omega}_j) \leq T_{\text{overlap}} \\ \infty & \text{otherwise} \end{cases}$$

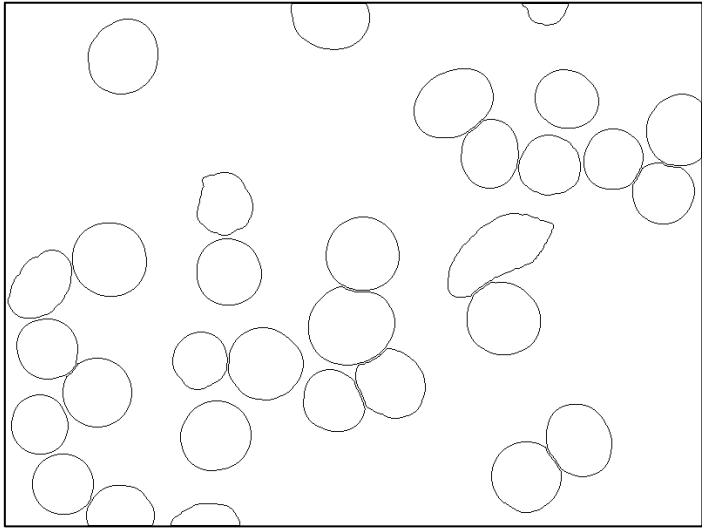
- Data Energy

$$U_d(\omega_i) = \begin{cases} 1 - \exp\left(-\frac{E(\hat{\omega}) - E_o}{3E_o}\right) & E(\hat{\omega}) \geq E_o \\ \left(\frac{E(\hat{\omega})}{E_o}\right)^{\frac{1}{3}} - 1 & \text{otherwise} \end{cases}$$

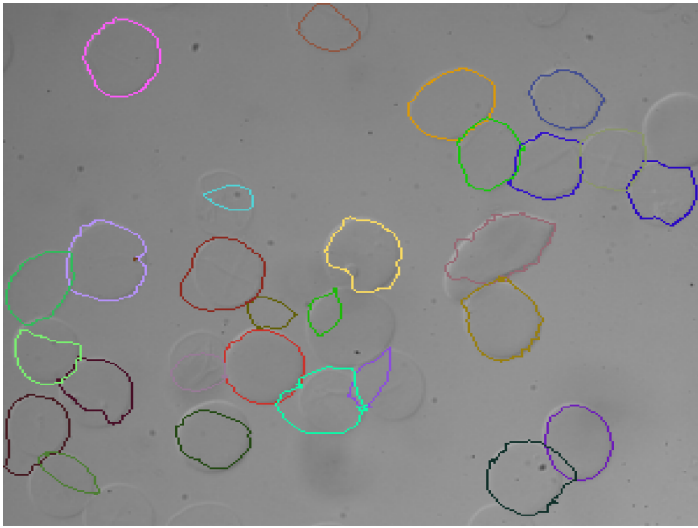
# Human red blood cells



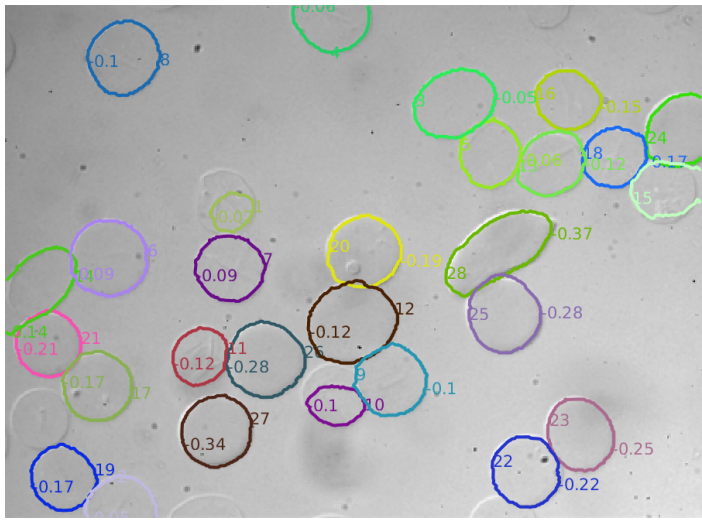
Original Image



Ground Truth



MPP-AC



MPP-LS



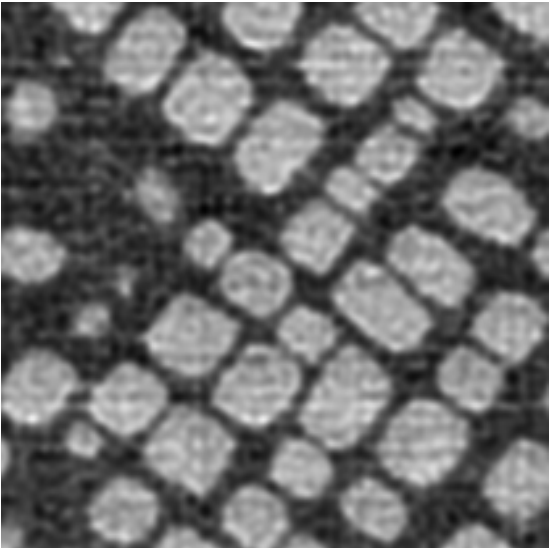
# F1 Scores

---

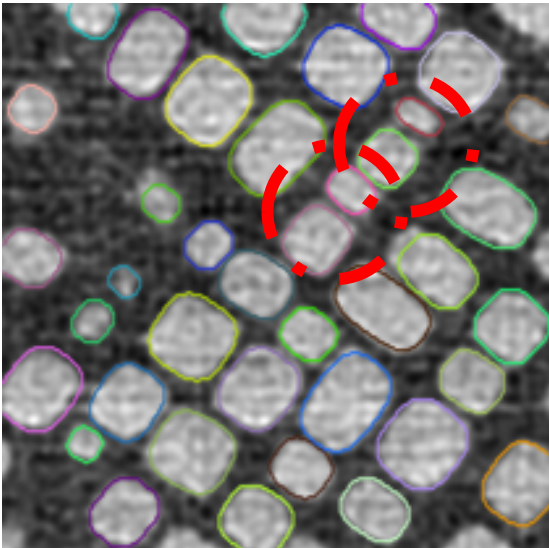
- Background
  - Pixelwise segmentation
- Outlines
  - Edges separated by a distance  $< 2$  pixels
- Counts
  - Intersection over union IOU  $> 80\%$

Method	Background	Outlines	Counts
MPP-AC	0.790	0.680	0.829
MPP-LS	0.843	0.820	0.916
Hybrid-LS	0.432	0.784	-

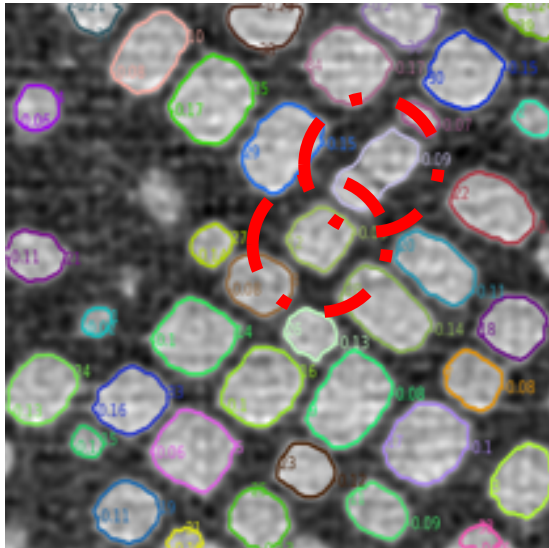
# NiCrAl Particles



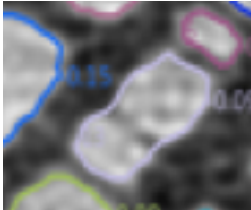
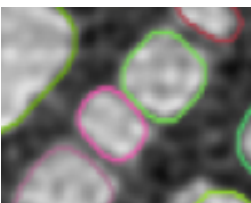
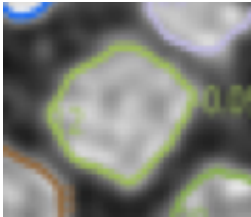
Original Image



MPP



MPP-LS

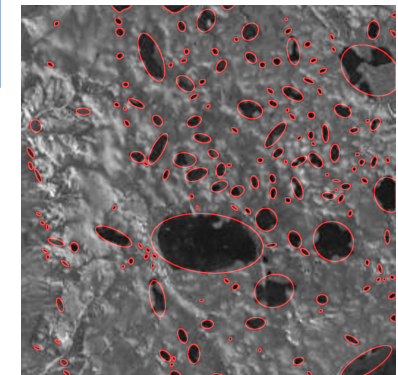
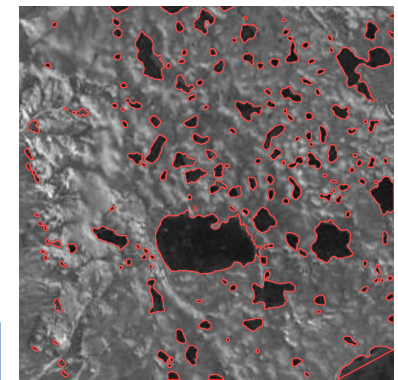
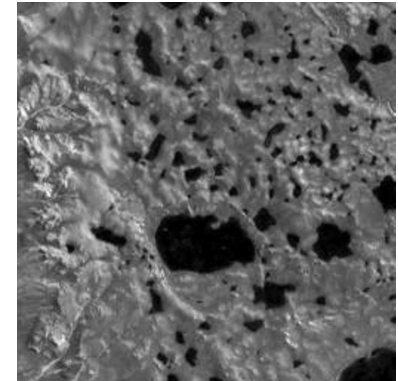
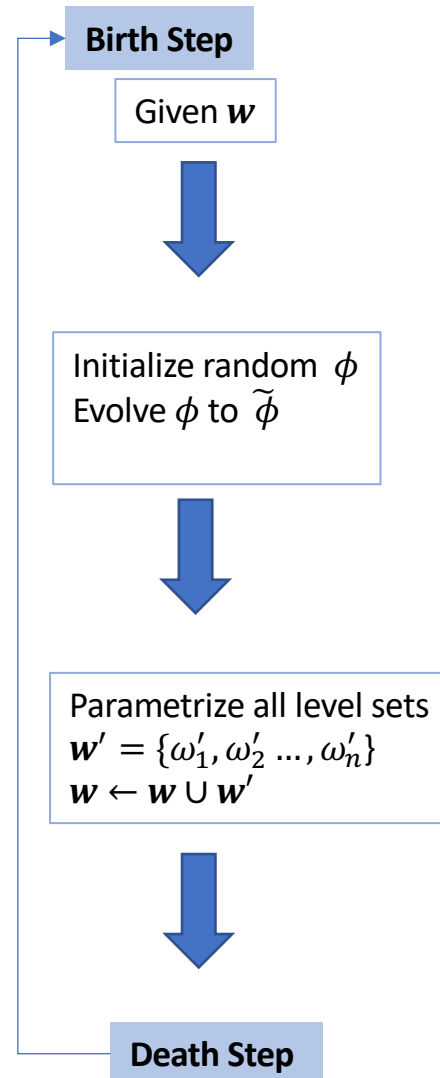


# Multiple Object Level Set

Goal: Use level sets to propose objects

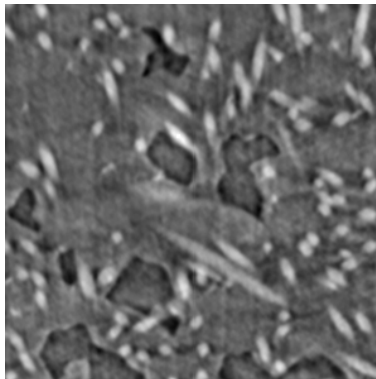
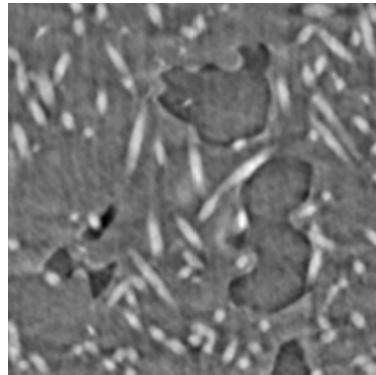
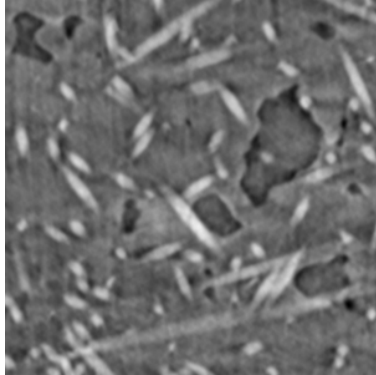
**Algorithm 2** Using Level Sets to simulate Multiple Births

```
1: procedure MPP ENERGY MINIMIZATION IN IMAGE  $K$ 
2:   Initialization:
3:   Initialize  $b_{rate} = b_o$ ,  $T = T_o$ ,  $\sigma = \sigma_o$ ,  $\mathbf{w} = \{\}$ 
4:   Birth Step:
5:   Initialize a level set  $\phi(k)$  at a random location
6:   Evolve  $\phi(k)$  to  $\tilde{\phi}(k)$ 
7:   Parametrize every closed contour  $\tilde{\omega}'$  in  $\tilde{\phi}(k)$ 
8:   Calculate a best fitting marked object  $\omega'$  for for each contour  $\tilde{\omega}'$ .
9:   Call  $\mathbf{w}' = \{\omega'_1, \omega'_2, \dots, \omega'_n\}$  the new configuration.
10:  Add the configuration to the current configuration  $\mathbf{w} \leftarrow \mathbf{w} \cup \mathbf{w}'$ 
11:  Death Step
12:  For every object  $\omega$  in  $\mathbf{w}$  calculate:
13:   $a_\omega = \exp \left[ \frac{U(\mathbf{w}) - U(\mathbf{w} - \omega)}{T^k} \right]$ , draw  $p$  form a uniform distribution over  $[0, 1]$ 
14:  if  $p < \frac{a_\omega \delta}{1 + a_\omega \delta}$ 
15:    remove  $\omega$  :  $\mathbf{w} \leftarrow \mathbf{w} - \omega$ 
16:  if  $n < Max\ Iterations$ 
17:    Update parameters:  $T^{k+1} \leftarrow T^k \times \alpha$ ,  $\sigma^{k+1} \leftarrow \sigma^k \times \alpha$ ,  $n \leftarrow n + 1$ 
18:    goto Birth Step
19: end procedure
```

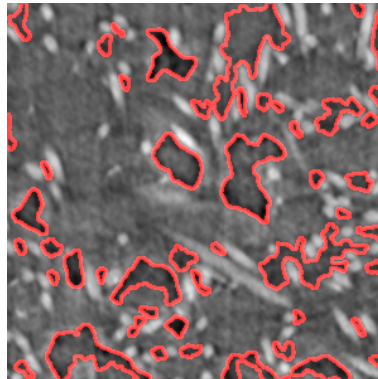
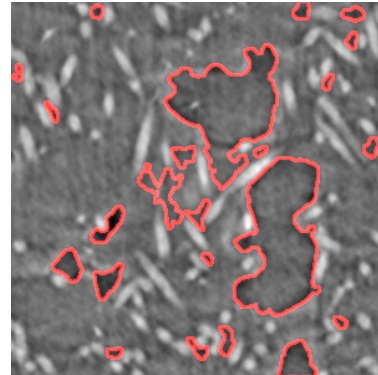
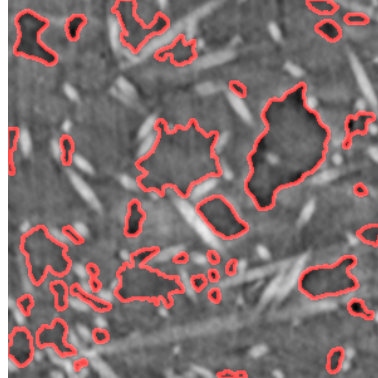


# Voids in fiber reinforced composites

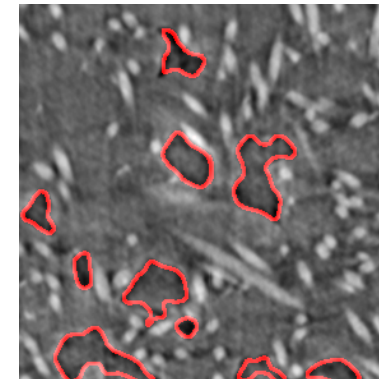
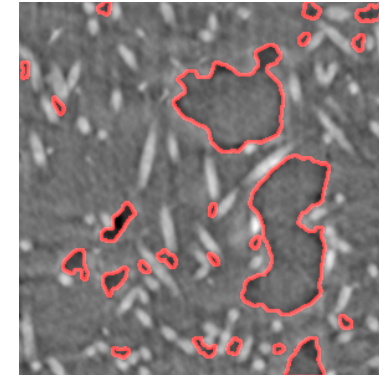
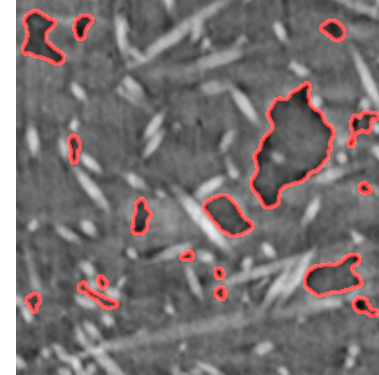
Original Image



Level Sets Only



MPP-LS



# Contributions of this work

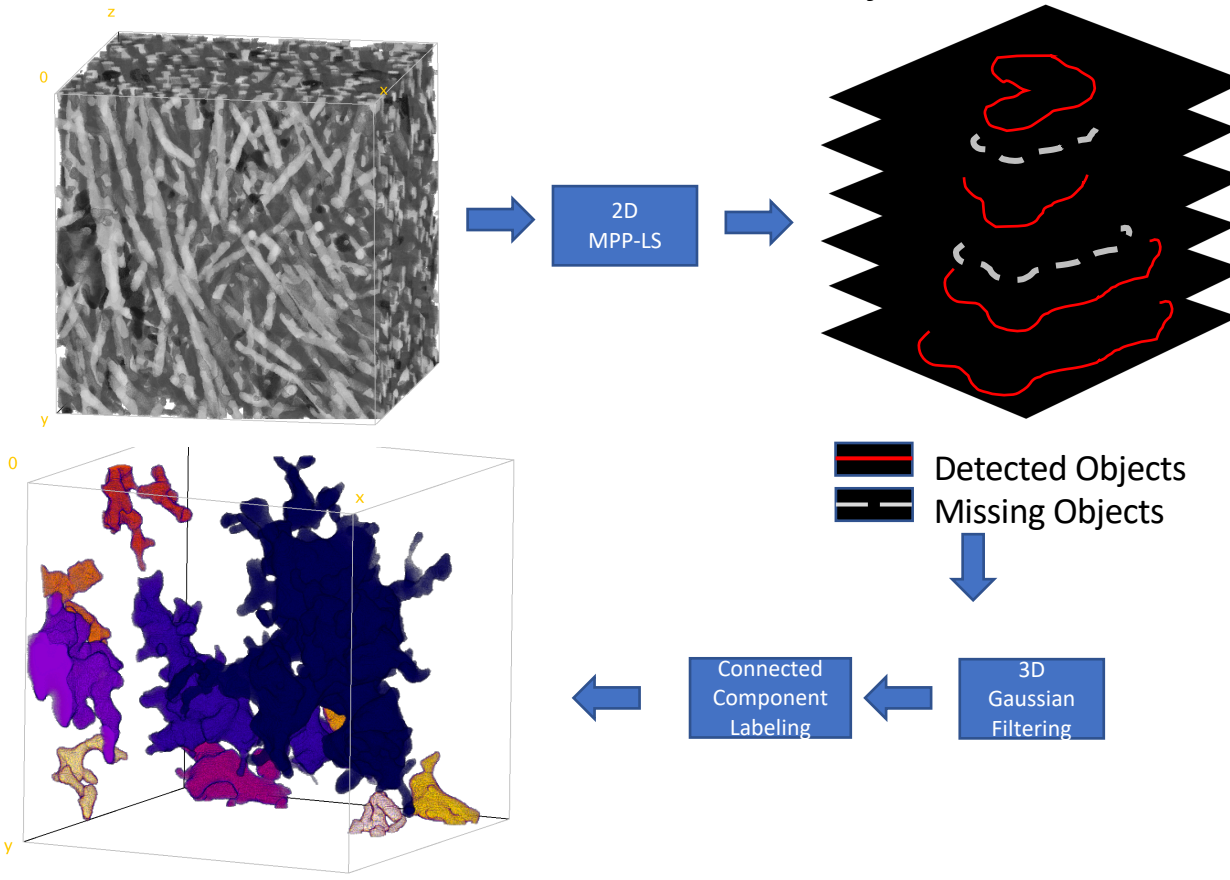
---

- Addition of the level sets method to the MPP model
- Extension of the a Hybrid level sets to incorporate a shape prior
- The use of level sets results to provide object proposals

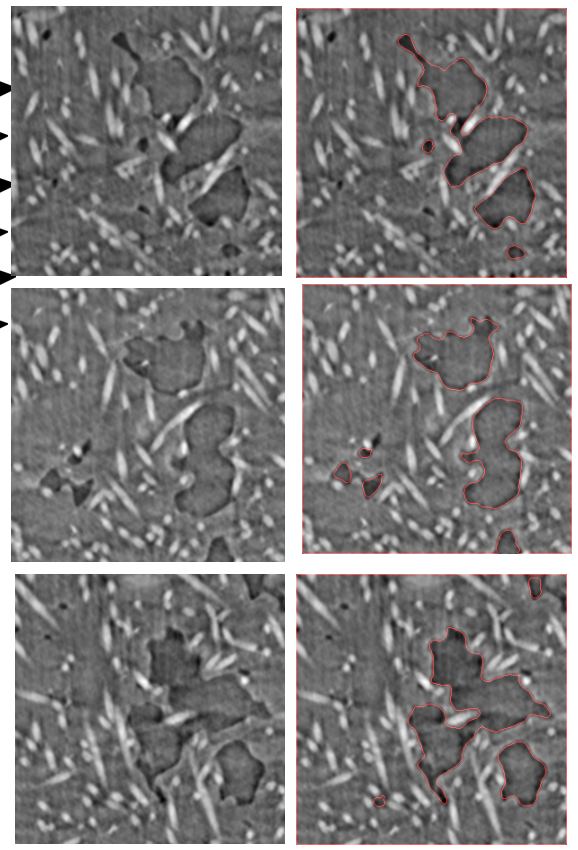
# Extension to 3D

# Use MPP-LS at each slice and filter

Procedure to obtain a 3D object



Example of 2D Cross Sections

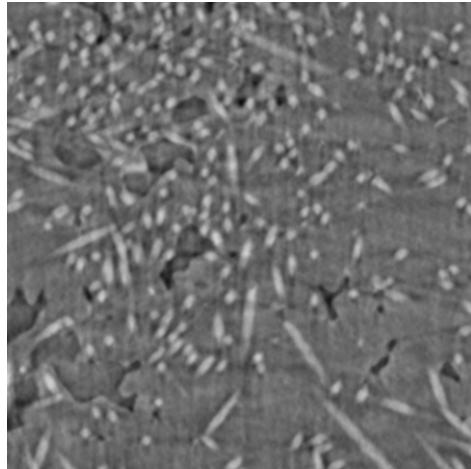


# Towards Deep Learning

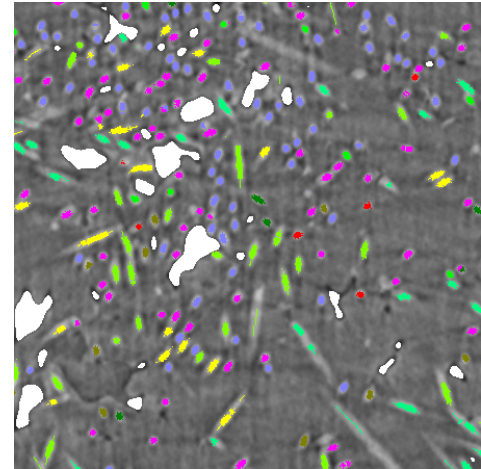


# U-Net beats its training data

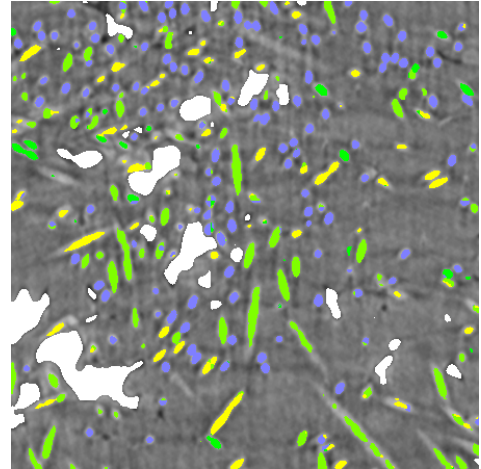
Original Image



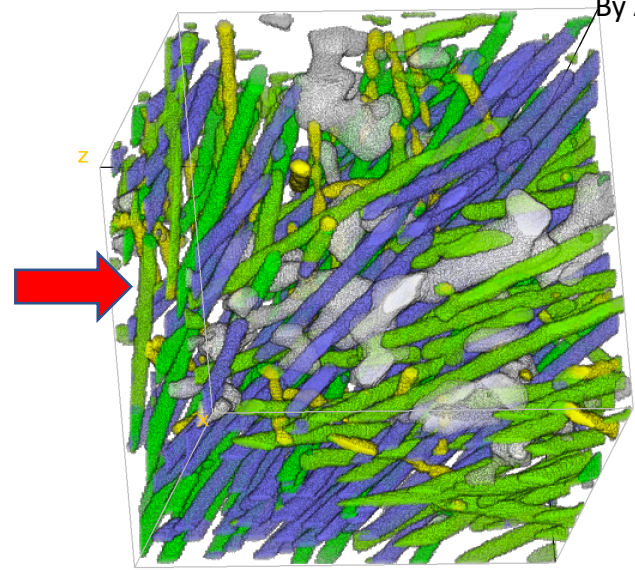
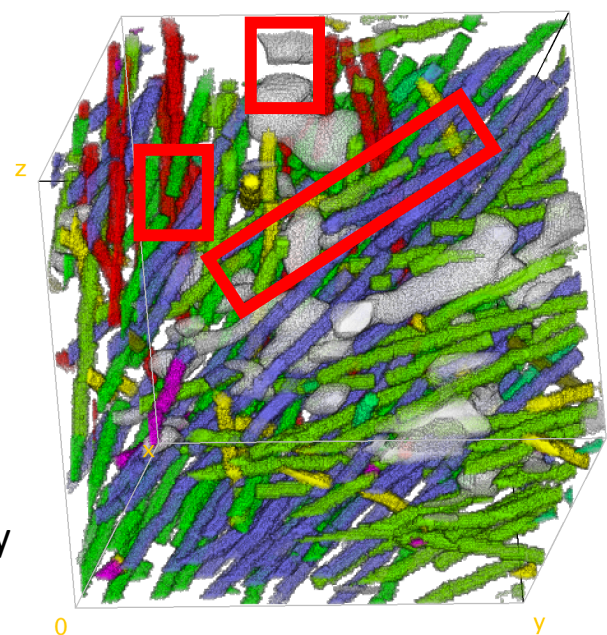
Training Data



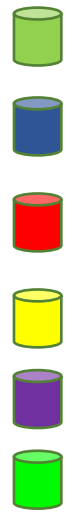
U-Net



- 3D U-Net Generalizes data extremely well
- 3D U-Net improves the results of its training data
- The GPU setup makes it significantly faster than MPP-LS

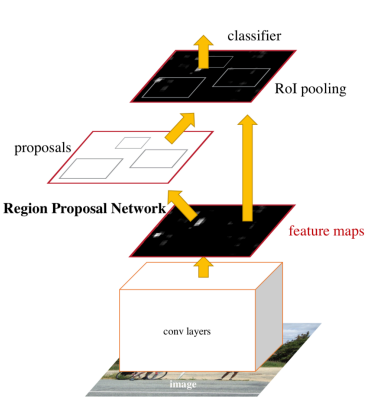


Fiber Class By Angles  $\phi, \theta$

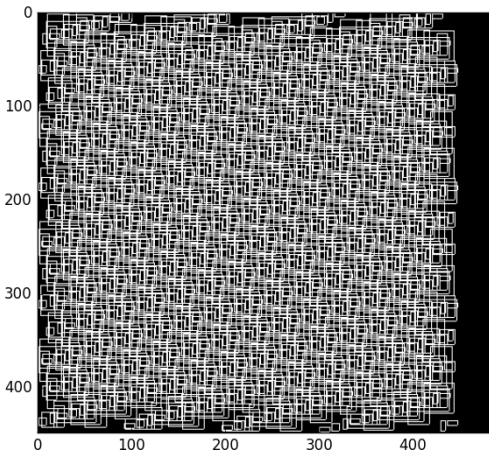


Corrections

# Faster RCNN could help guiding MCMC



Faster-RCNN

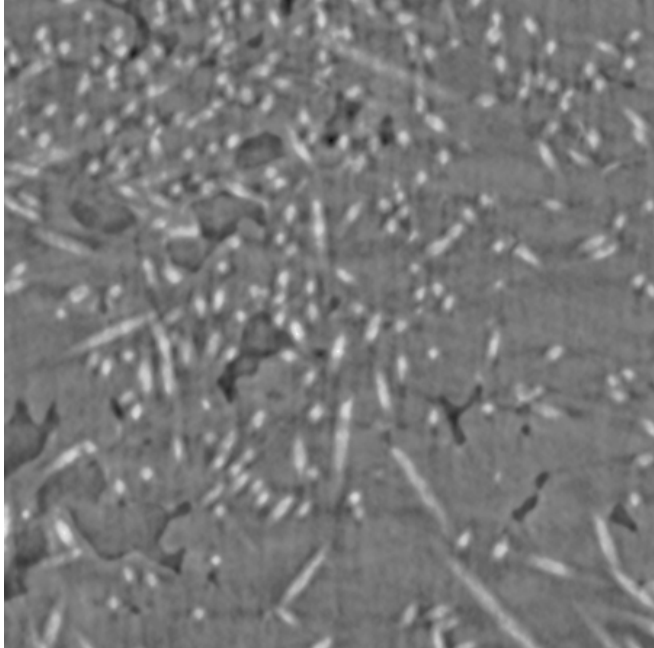


Anchor Proposals

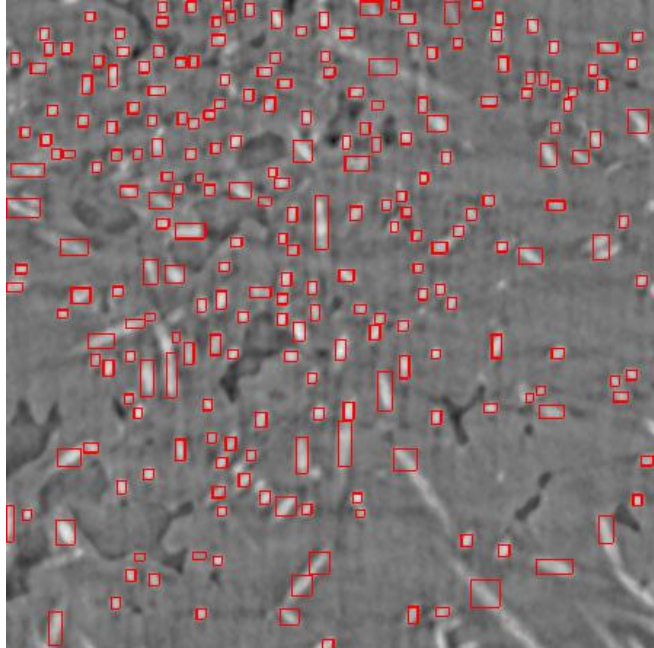
Fast RCNN has parallel relations with RJMCMC:

- Anchor Proposal  $\equiv$  birth death process
- Bounding Box Refinement  $\equiv$  perturbations

Original Image



Faster-RCNN Results



# References

- M. Kulikova, I. Jermyn, X. Descombes, E. Zhizhina, and J. Zerubia, “Extraction of arbitrarily-shaped objects using stochastic multiple birth-and-death dynamics and active contours,” *Electronic Imaging*, vol. 7533, 2010
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- M. L. Comer and E. J. Delp, “The EM/MPM algorithm for segmentation of textured images: Analysis and further experimental results,” *IEEE Transactions on Image Processing*, vol. 9, no. 10, pp. 1731–1744, 2000.
- Kass, M., Witkin, A. & Terzopoulos, “Snakes: Active contour models,” *D. Int J Comput Vision* (1988) 1: 321
- X. Descombes, R. Minlos, and E. Zhizhina, “Object extraction using a stochastic birth-and-death dynamics in continuum,” *Journal of Mathematical Imaging and Vision*, vol. 33, no. 3, pp. 347–359, Mar 2009.
- Z. Yan, B. J. Matuszewski, L. K. Shark, and C. J. Moore, “Medical image segmentation using new hybrid level-set method,” *Proceedings - 5th International Conference BioMedical Visualization, Information Visualization in Medical and Biomedical Informatics, MediVis 2008*, no. 1, pp. 71–76, 2008.
- Osher, S. and Sethian, J.A., *Fronts Propagating with Curvature Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulations*, *J. Comput. Phys.* 79, 12-49 (1988).

# Publications from this work

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- C. Aguilar and M. Comer, "A Marked Point Process Model Incorporating Active Contours Boundary Energy," *Electronic Imaging*, vol. 2018, no. 15, pp. 230-2304, 2018
- \*(draft) C. Aguilar and M. Comer, "Combining Level sets in the Marked Point Process Framework," *International Symposium on Visual Computing (ISVC)*. July, 2019.

# Summary of contributions

---

- Exploration of the MPP combined with:
  - Parametric active contours
  - Level sets
- We used multiple birth and death to sample our space but we also explored using only the level set results
- We obtained preliminary 3D data and trained Neural Networks with this data.

Thanks